15-295 Spring 2019 Problem Set 5 Flows and Matchings

A. Book Club

Porto's book club is buzzing with excitement for the annual book exchange event! Every year, members bring their favorite book and try to find another book they like that is owned by someone willing to trade with them.

I have been to this book exchange before, and I definitely do not want to miss it this year, but I feel that the trading should be improved. In the past, pairs of members interested in each other's books would simply trade: imagine that person A brought a book that person B liked and vice-versa, then A and B would exchange their books.



The famous staircase in Lello bookstore, downtown Porto.

I then realized that many members were left with the same book they walked-in with... If instead of looking for pairs I looked for triplets, I could find more valid exchanges! Imagine that member A only likes member B's book, while B only likes C's book and C likes A's book. These 3 people could trade their books in a cycle and everyone would be happy!

But why stop at triplets? Cycles could be bigger and bigger! Could you help me find if it is possible for everyone to go out with a new book? Be careful, because members will not give their book without receiving one they like in return.

Task

Given the members of the book club and the books they like, can we find cycles so that everyone receives a new book?

Input

The first line has two integers: N, the number of people, and M, the total number of "declarations of interest". Each of the following M lines has two integers, A and B, indicating that member A likes the book that member B brought $(0 \le A, B < N)$. Numbers A and B will never be the same (a member never likes the book he brought).

Output

You should output YES if we can find a new book for every club member and NO if that is not possible.

Constraints

 $\begin{array}{l} 2 \leq N \leq 10\,000 \\ 1 \leq M \leq 20\,000 \mbox{ and } M \leq N^2 - N. \end{array}$

Sample Input

99 01

1 2

2 0

34

43

56

67

78

85

Sample Output

YES

Problem B Elementary Math Time limit: 10 seconds

Ellen is teaching elementary math to her students and the time for the final exam has come. The exam consists of n questions. In each question the students have to add (+), subtract (-) or multiply (*) a pair of numbers.

Ellen has already chosen the n pairs of numbers. All that remains is to decide for each pair which of the three possible operations the students should perform. To avoid students getting bored, Ellen wants to make sure that the n correct answers to her exam are all different.

Please help Ellen finish constructing the exam by automating this task.



Example exam by Ellen

Input

The input consists of:

- one line with one integer $n \ (1 \le n \le 2500)$, the number of pairs of numbers;
- *n* lines each with two integers *a* and *b* ($-10^6 \le a, b \le 10^6$), a pair of numbers used.

Output

For each pair of numbers (a, b) in the same order as in the input, output a line containing a valid equation. Each equation should consist of five parts: a, one of the three operators, b, an equals sign (=), and the result of the expression. All the n expression results must be different.

If there are multiple valid answers you may output any of them. If there is no valid answer, output a single line with the string "impossible" instead.

Sample Input 1	Sample Output 1
4	1 + 5 = 6
1 5	3 * 3 = 9
3 3	4 - 5 = -1
4 5	-16 = 5
-1 -6	

Sample Output 2

4	impossible
-4 2	
-4 2	
-4 2	
-4 2	

C. Single-use Stones

1 second, 256 megabytes

A lot of frogs want to cross a river. A river is w units width, but frogs can only jump l units long, where l < w. Frogs can also jump on lengths shorter than l. but can't jump longer. Hopefully, there are some stones in the river to help them.

The stones are located at integer distances from the banks. There are a_i stones at the distance of i units from the bank the frogs are currently at. Each stone can only be used once by one frog, after that it drowns in the water.

What is the maximum number of frogs that can cross the river, given that then can only jump on the stones?

Input

The first line contains two integers w and l ($1 \le l < w \le 10^5$) – the width of the river and the maximum length of a frog's jump.

The second line contains w - 1 integers $a_1, a_2, ..., a_{w-1}$ ($0 \le a_i \le 10^4$), where a_i is the number of stones at the distance *i* from the bank the frogs are currently at.

Output

Print a single integer — the maximum number of frogs that can cross the river.

nput
) 5 0 1 0 2 0 0 1 0
utput
nnut
)
utput

In the first sample two frogs can use the different stones at the distance 5, and one frog can use the stones at the distances 3 and then 8.

In the second sample although there are two stones at the distance 5, that does not help. The three paths are: $0 \rightarrow 3 \rightarrow 6 \rightarrow 9 \rightarrow 10, 0 \rightarrow 2 \rightarrow 5 \rightarrow 8 \rightarrow 10, 0 \rightarrow 1 \rightarrow 4 \rightarrow 7 \rightarrow 10$.

3

8

 $0 \rightarrow 3 \rightarrow 6 \rightarrow 9 \rightarrow 10 \ 0 \rightarrow 2 \rightarrow 5 \rightarrow 8 \rightarrow 10 \ 0 \rightarrow 1 \rightarrow 4 \rightarrow 7 \rightarrow 10$

D. Delivery Bears

2 seconds, 256 megabytes

Niwel is a little golden bear. As everyone knows, bears live in forests, but Niwel got tired of seeing all the trees so he decided to move to the city.

In the city, Niwel took on a job managing bears to deliver goods. The city that he lives in can be represented as a directed graph with n nodes and m edges. Each edge has a weight capacity. A delivery consists of a bear carrying weights with their bear hands on a simple path from node 1 to node n. The total weight that travels across a particular edge must not exceed the weight capacity of that edge.

Niwel has **exactly** *x* bears. In the interest of fairness, no bear can rest, and the weight that each bear carries must be exactly the same. However, each bear may take different paths if they like.

Niwel would like to determine, what is the maximum amount of weight he can deliver (it's the sum of weights carried by bears). Find the maximum weight.

Input

The first line contains three integers *n*, *m* and *x* ($2 \le n \le 50$, $1 \le m \le 500$, $1 \le x \le 100\ 000$) — the number of nodes, the number of directed edges and the number of bears, respectively.

Each of the following *m* lines contains three integers a_i , b_i and c_i ($1 \le a_i$, $b_i \le n$, $a_i \ne b_i$, $1 \le c_i \le 1\ 000\ 000$). This represents a directed edge from node a_i to b_i with weight capacity c_i . There are no self loops and no multiple edges from one city to the other city. More formally, for each *i* and *j* that $i \ne j$ it's guaranteed that $a_i \ne a_j$ or $b_i \ne b_j$. It is also guaranteed that there is at least one path from node 1 to node *n*.

Output

Print one real value on a single line — the maximum amount of weight Niwel can deliver if he uses exactly x bears. Your answer will be considered correct if its absolute or relative error does not exceed 10^{-6} .

Namely: let's assume that your answer is a, and the answer of the jury is b. The checker program will consider your answer correct if $\frac{|a-b|}{max(1,b)} \leq 10^{-6}$.

Input	
4 3	
2 2	
4 1	
31	
4 2	
putput	
.500000000	

input
5 11 23
1 2 3
2 3 4
3 4 5
4 5 6
134
2 4 5
3 5 6
1 4 2
2 5 3
1 5 2
3 2 30
output
10.222222222

In the first sample, Niwel has three bears. Two bears can choose the path $1 \rightarrow 3 \rightarrow 4$, while one bear can choose the path $1 \rightarrow 2 \rightarrow 4$. Even though the bear that goes on the path $1 \rightarrow 2 \rightarrow 4$ can carry one unit of weight, in the interest of fairness, he is restricted to carry 0.5 units of weight. Thus, the total weight is 1.5 units overall. Note that even though Niwel can deliver more weight with just 2 bears, he must use exactly 3 bears on this day.

E. Vasya And The Matrix

2 seconds, 256 megabytes

Now Vasya is taking an exam in mathematics. In order to get a good mark, Vasya needs to guess the matrix that the teacher has constructed!

Vasya knows that the matrix consists of *n* rows and *m* columns. For each row, he knows the xor (bitwise excluding or) of the elements in this row. The sequence $a_1, a_2, ..., a_n$ denotes the xor of elements in rows with indices 1, 2, ..., *n*, respectively. Similarly, for each column, he knows the xor of the elements in this column. The sequence $b_1, b_2, ..., b_m$ denotes the xor of elements in columns with indices 1, 2, ..., *m*, respectively.

Help Vasya! Find a matrix satisfying the given constraints or tell him that there is no suitable matrix.

Input

The first line contains two numbers *n* and $m (2 \le n, m \le 100)$ — the dimensions of the matrix.

The second line contains *n* numbers $a_1, a_2, ..., a_n$ ($0 \le a_i \le 10^9$), where a_i is the xor of all elements in row *i*.

The third line contains *m* numbers $b_1, b_2, ..., b_m$ ($0 \le b_i \le 10^9$), where b_i is the xor of all elements in column *i*.

Output

If there is no matrix satisfying the given constraints in the first line, output "NO".

Otherwise, on the first line output "YES", and then *n* rows of *m* numbers in each $c_{i1}, c_{i2}, \ldots, c_{im}$ $(0 \le c_{ij} \le 2 \cdot 10^9)$ – the description of the matrix.

If there are several suitable matrices, it is allowed to print any of them.

put	
13	
tput	
5 8	
put	
6 5 12	
tput	

F. Recover a functional graph

2 seconds, 256 megabytes

Functional graph is a directed graph in which all vertices have outdegree equal to 1. Loops are allowed.

Some vertices of a functional graph lay on a cycle. From the others we can come to a cycle by making a finite number of steps along the edges (we consider only finite functional graphs in this problem).

Let's compute two values for each vertex. $precycle_i$ is the amount of edges we should pass to get to a vertex which is a part of some cycle (zero, if *i* itself lies on a cycle), $cycle_i$ is the length of the cycle we get to.

You are given the information about these values for some functional graph. For each vertex you know the values $precycle_i$ and $cycle_i$, however, instead of some values there can be the question mark. It means that these values are unknown.

Build any functional graph that suits the description or determine that there is no such graph.

Input

The first line contains single integer n ($1 \le n \le 300$) — the number of vertices in the graph.

Each of the next *n* lines contain two integers $- precycle_i$ ($0 \le precycle_i \le n - 1$) and $cycle_i$ ($1 \le cycle_i \le n$). There could be question marks instead of some of these values.

Output

In case there is no solution, print -1.

Otherwise, print n integers. i-th of them is the number of vertex to which the edge form the i-th vertex go.

The vertices should be in the same order as they go in input data.

If there are multiple solutions, print any of them.

input	
3	
0 3	
0 3	
5 5	
output	
2 3 1	

input	
5	
? ? ? ?	More sample input and output
5.5	available online.
; ;	
output	
5 3 2 2 4	