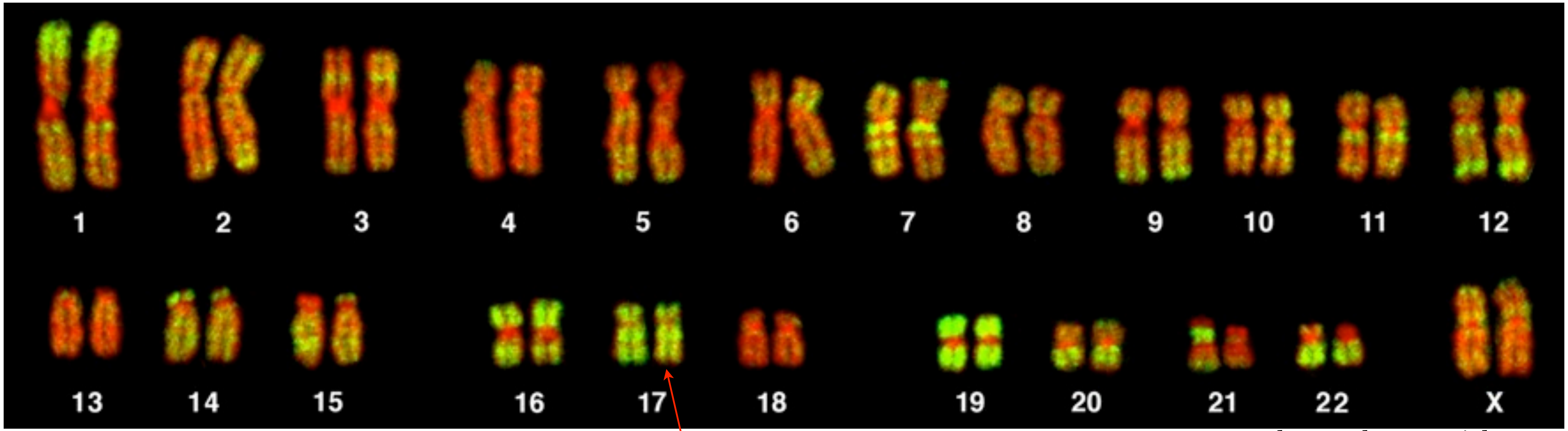
# String Matching Z

(Following Gusfield Chapter 2)

# Exact String Search

Microscopy of chromosomes of a human female (karyotype):



Bolzer et al, PLoS Biol. 2005

`ggccgggccctgtgaccacagtccacatcacaccaggacacagaggaagggccgggccctgtgaccacagtccacatcacaccaggacacagaggaagggccgggcctcatgaccacagt`

`gtccacatcaca`

Where does this string occur in the genome?

# Exact String Matching

**Exact String Matching Problem**. Given a (long) string $T$ and a shorter string $P$, find all occurrences of $P$ in $T$. Occurrences of $P$ are allowed to overlap.

- Motivation is obvious:
  - search for words in long documents, webpages, etc.
  - find subsequences of DNA, proteins that are known to be important.

- We'll see 4 efficient algorithms for this problem.

# The Simple (Slow) Algorithm

```
SimpMatch(T, P):
  for i = 1..|T|:
    j = 1
    while j ≤ |P| and T[i+j-1] == P[j]:
      j += 1
    if j == |P|+1: print “Occurs at”, i
```

- Runs in O($|T| \times |P|$) time.
- Information gathered in **while** loop at iteration *i* is ignored in iteration *i*+1.
- Key idea for speeding it up: use what we learned about *T* in the **while** loop to increment *i* by more than 1 in the outer loop.

# Exploiting Patterns in P

```
         i
         ↓
All this happened, more or less.
         happy
          happy
```

- After comparing “happy” to “happe” at iteration $i$,
  - we know that $T[i \ldots i+3]$ = “happ” = $P[1 \ldots 4]$
  - we can deduce that there can be no match at $i+1$ because $T[i+1]$ = P[2] = “a” but $P[1]$ = “h”
  - in fact, since “h” does not appear in $T[i \ldots i+3] = P[1 \ldots 4]$, we could set $i = i + 4$

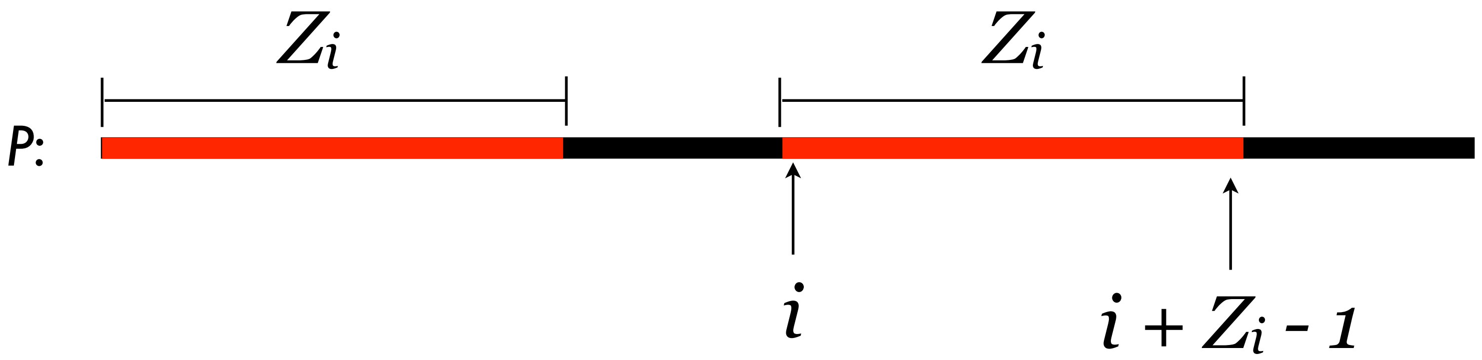
- Since $T$ will have matched some part of $P$, it is the similarities between various parts of $P$ that allow us to make these deductions.

$\Longrightarrow$ Preprocess $P$ to find these similarities.

# Z-Algorithm

# Fundamental Preprocessing

**Def.** $Z_i(P)$ = the length of the longest substring of $P$ that starts at $i > 1$ and matches a prefix of $P$.



- $P$ = “aardvark”: $Z_2 = 1$, $Z_6 = 1$
- $P$ = “alfalfa”: $Z_4 = 4$
- $P$ = “photophosphorescent”: $Z_6 = Z_{10} = 3$

# String Search With $Z_i$

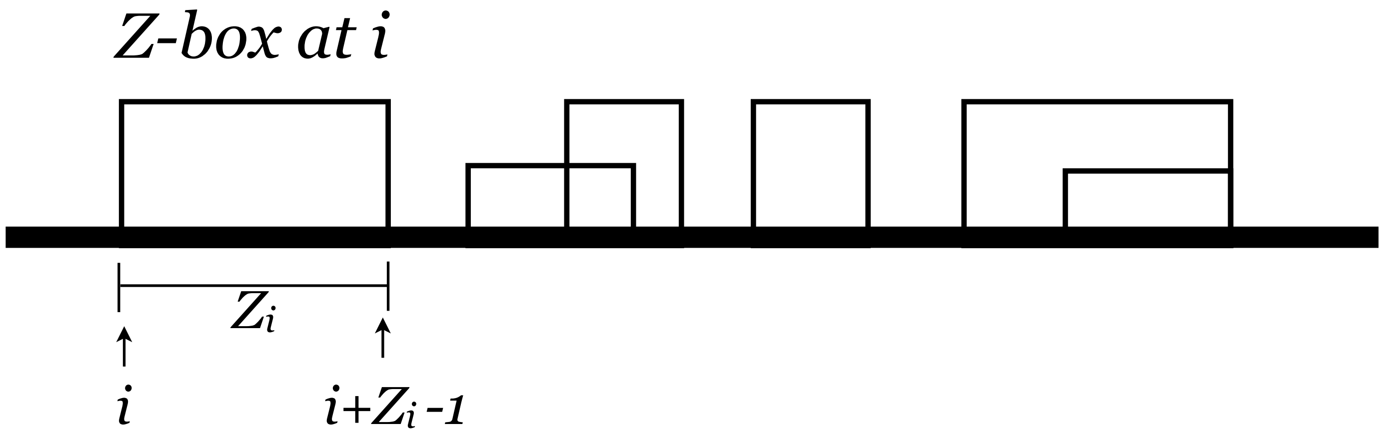
```
ZMatch(T, P):
  S = P$T
  Compute all $Z_i$ for S
  return all i-|P|-1 such that $Z_i$ = |P|
```

(map indices of $S$ to indices of $T$)

Why does this work?

- $Z_i = |P|$ if and only if the string starting at $i$ matches $P$.
- Running time is $O(|P| + |T| + Z_S)$, where $Z_S$ is the time to compute the $Z_i$ for $T$.
- **Next**: an $O(|P| + |T|)$ algorithm for computing the $Z_i$.

# Z Boxes



**Def.** *Z-box at i* is the substring starting at *i* and continuing to $i+Z_i-1$. This is the substring that matches the prefix. There is no Z-box at *i* if $Z_i = 0$.

- Algorithm for computing $Z_i$ will iteratively compute $Z_k$ given:
  - $Z_2 \ldots Z_{k-1}$, and
  - the boundaries *l*, *r* of the rightmost Z-box found starting someplace in $2 \ldots k-1$.

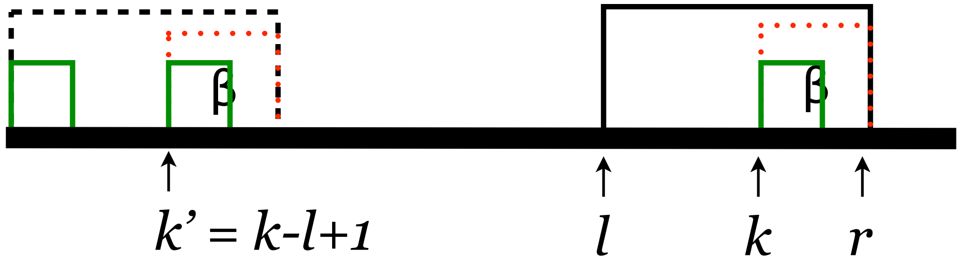
# Z Algorithm

- Input: $Z_2 \ldots Z_{k-1}$, and the boundaries $l$, $r$ of the rightmost Z-box found starting someplace in $2 \ldots k-1$.
- Output: $Z_k$, and updated $l$, $r$

1. If $k > r$, explicitly compute $Z_k$ by comparing with prefix.
   If $Z_k > 0$: $r = k + Z_k - 1$ and $l = k$ (since this is a new farther right Z-box).
2. If $k \leq r$, this is the situation:

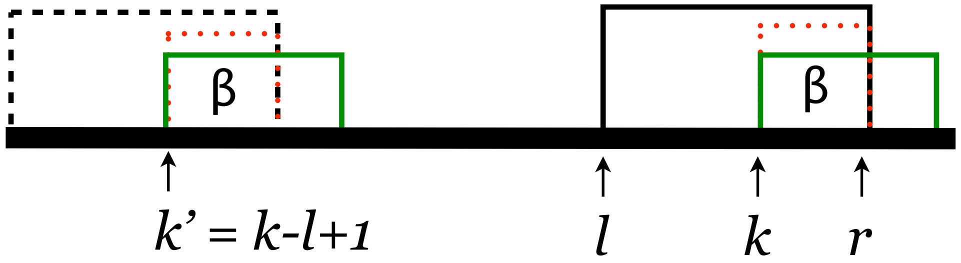
Two subcases:

$Z_{k'} < \beta$:



Set $Z_k = Z_{k'}$ and leave $l$, $r$ unchanged.

$Z_{k'} \geq \beta$:



Explicitly compare after $r$ to set $Z_k$.
$l = k$, $r$ = point where comparison failed

# Analysis

- Runs in O(|$P$|) time:
    - only match characters covered by a Z-box once, so there are O(|$P$|) matches.
    - every iteration contains at most one mismatch, so there are O(|$P$|) mismatches.
- Correctness follows by induction and the arguments we made in the description of the algorithm.
- Immediately gives an O(|$P$| + |$T$|)-time algorithm for string matching as described a few slides ago.
    - O(|$P$| + |$T$|) is the best possible worst-case running time, since you might have to look at the whole input.
    - But better algorithms exist in practice that, for real instances, have expected sublinear runtime.